LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2007

ST 3501 - STATISTICAL MATHEMATICS - II

BB 22

Date : 27/10/2007 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART – A

Answer ALL questions. **(10 x 2 = 20 marks)**

1. Define Riemann integral.
2. Give an example of a bounded function, which is not Riemann integrable over [0,1].
3. Find the Laplace transform of $\frac{\cos 3t - \cos 2t}{t}$
4. Let X be a continuous random variable with p.d.f.

$$f(x) = \frac{1}{\pi(1+x^2)}, \; -\infty < x < \infty$$

Show that expectation of X does not exist.

5. Solve the differential equation

$$xdx + ydy = a(x^2 + y^2)dy$$

6. Solve the differential equation

$$(D^2 - 3D + 2)y = e^{5x} + 2$$

7. Show that the system of equations

$$3x - 4y = 2;\; 5x + 2y = 12;\; -x + 3y = 1$$

λs *consistent*

8. Verify Cayley – Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$
9. If the two dimensional random variable (X, Y) has the joint p.d.f

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), 0 < x < 1, 0 < y < 1 \\ o \quad , \quad otherwise, \end{cases}$$

find the marginal p.d.f of Y.

10. Define variance – Covariance matrix of a random vector.

PART – B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. Let $f(x) = x$ for $0 \le x \le 1$ and $\sigma_n \left\{0, \frac{1}{n}, \frac{2}{n}, ..., 1\right\}$ be a partition of [0,1]. Compute $\underset{n\to\infty}{lt}\, U[f;\sigma_n]$ and $\underset{n\to\infty}{lt}\, L[f;\sigma_n]$.
12. For a random variable X, E(x) = 10 and V(x) = 25. Find the positive values of a and b such that Y=ax-b has expectation zero and variance 1.
13. The p.d.f of a continuous random variable is given as

$$f(x) =_{\frac{1}{2}} e^{-1x1}, \quad -\infty < x < \infty$$

Find M.G.F of x and hence find the mean and variance of x.

14. Solve the following differential equation using Laplace Transform.

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-t} \text{ given that y=0, } \frac{dy}{dt} = o \text{ when t=0}$$

15. Show that the following system of equations is consistent and hence solve them.

$$x = 2y - z = 3; \quad 3x - y + 2z = 1$$

$$2x - 2y + 3z = 2; \quad x - y + z = -1$$

16. Using the method of Laplace transform solve,

$$\frac{dx}{dt} + 2x - 3y = 2t$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$

given x(0)=0 and y(0)=0.

17. The joint p.d.f of the random variables x and y is given by,

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0 \\ 0 & , \ otherwise \end{cases}$$

Find the marginal p.d.f's of x and y Also find $C_oV(X,Y)$.

18. Prove that the matrices A,B and C given below have the same characteristic roots.

$$A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}; \qquad B = \begin{bmatrix} 0 & b & a \\ b & 0 & c \\ a & c & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & c & b \\ c & 0 & a \\ b & c & 0 \end{bmatrix}$$

PART – C

Answer any TWO questions. **(2 x 20 = 40 marks)**

19. a) State and prove the first fundamental theorem of integral calculus.

b) If the moments of x are defined by

$E[x^r] = 0.6$ for r=1,2,3, …….

Show that P(x=0)=0.4; P(x=1)=0.6

P[x=x]=0, otherwise.

20. a) Find the Laplace transforms of the following functions.

(i) $\frac{\sin^2 t}{t}$ (ii) $\cos^2 3t - \cos^2 2t$

b) Evaluate the following integrals.

(i) $\int_0^\infty \frac{xdx}{1+x^b}$ (ii) $\int_0^\infty e^{-x^2} dx$

21. a) The joint p.d.f of the random variable (X,Y) is

$$f(x, y) = \begin{cases} \frac{1}{4} e^{\frac{-(x-y)}{2}}, & x > 0, \ y > 0 \\ 0 & , \ otherwise \end{cases}$$

Find the distribution of $\frac{X - Y}{2}$

22. a) Find all the characteristic roots and the associated characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b) Show that the system of equations

$$x = 2y - z = 3 \ ; \qquad 3x - y + 2z = 1$$

$$2x - 2y + 3z = 2 \ ; \qquad x - y + z = -1$$

is consistent and solve them.

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